DE-3869

Sub. Code
11

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

ALGEBRA

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) Prove that HK is a subgroup of G if and only if HK=KH.

   (b) If H and K are finite subgroups of a group G then show that $O(HK) = \frac{O(H)\,O(K)}{O(H \cap K)}$.

2. (a) Define a P-Sylow subgroup of G and also show that the number of P-Sylow subgroup in G for a given prime is of the form 1 + KP.

   (b) Show that any group of order $11^2.13^2$ must be abelian.

3. (a) If R is a commutative ring with unit element and M is an ideal of R, then prove that M is a maximal ideal of R if and only if R/M is a field.

   (b) Show that every finite integral domain is a field.

4. (a) State and prove unique factorization theorem.

   (b) Show that if R is a unique factorization domain then so is $R[x]$.

5. (a) Prove that any two basis of a finite dimensional Vector space V have the same number of elements.

   (b) If V is finite dimensional and if W is a subspace of V, then show that W is finite dimensional, dim W $\leq$ dim V and dim $V/_M = \text{dim}V - \text{dim}W$.

6. (a) Prove that if $T \in A(V)$ has all its characteristic roots in F then there is a basis of V in which the matrix of T is triangular.

   (b) If $S,T \in A(V)$ then prove that

   (i) $T^* \in A(V)$

   (ii) $(T^*)^* = T$

   (iii) $(S+T)^* = S^* + T^*$

   (iv) $(\lambda S)^* = \bar{\lambda} S^*$ for $\lambda \in F$

   (v) $(ST)^* = T^* S^*$

7. (a) Prove that the element $a \in k$ is algebraic over F if and only if F(a) is a finite extension of F, Where F(a) is the smallest subfield K containing F and a.

   (b) Show that a polynomial of degree n over a field can have atmost n roots in any extension field.

8. (a) Prove that for every prime number P and every positive integer m there exists a field having p^m elements.

   (b) Show that if F is a finite field and $\alpha \neq 0$, $\beta \neq 0$ are two elements of F then we can find elements a, b in F such that $1 + \alpha a^2 + \beta b^2 = 0$.

DE–3870

Sub. Code
12

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

REAL ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries equal marks.

1. (a) State and prove Bolzano-Weierstrass theorem. (10)

   (b) If $\{k_\alpha\}$ is a collection of compact sets in a metric space x such that the intersection of every finite subcollection of $\{k_\alpha\}$ is non empty, then prove that $\bigcap k_\alpha \neq \phi$. (10)

2. (a) Prove that every k-cell is compact. (10)

   (b) Prove that subset E of the real line $\mathbb{R}$ is connected if and only if E has the following property :

   If $x, y \in E$ and $x < z < y$ then $z \in E$. (10)

3. (a) State and prove Rolle's theorem. (10)

   (b) State and prove generalized mean-value theorem. (10)

4. State and prove implicit function theorem. (20)

5. (a) If f is monotonic on $[a, b]$ and if α is continuous on $[a, b]$ the prove that $f \in k(\alpha)$. (10)

(b) Suppose $f \in k(\alpha)$ on $[a, b]$, $m \leq f \leq M, \phi$ is continuous on $[m, M]$ and $h(x) = \phi(f(x))$ on $[a, b]$. Then prove that $h \in k(\alpha)$ on $[a, b]$. (10)

6. (a) Suppose $c_n \geq 0$, for $n = 1, 2, 3, ... \sum c_n$ converges, $\{s_n\}$ is a sequence of distinct points in (a, b) and $\alpha(x) = \sum_{n=1}^{\infty} c_n \, I(x - s_n)$. If f be continuous on $[a, b]$ then prove that $\int_a^b f \, d\alpha = \sum_{n=1}^{\infty} c_n f(s_n)$. (10)

(b) State and prove the fundamental theorem of calculus. (10)

7. (a) State and prove the countably subadditive property. (10)

(b) Prove that the family M of measurable sets is an algebra of sets. (10)

8. (a) State and prove Fatou's lemma. (10)

(b) State and prove Lebergue dominated convergence theorem. (10)

DE–3871

Sub. Code
13

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

DIFFERENTIAL EQUATIONS AND NUMERICAL METHODS

Time : Three hours　　　　Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) (i) Consider the equation $y'' + y' - 6y = 0$. Find the solution ϕ satisfying $\phi(0) = 1$, $\phi'(0) = 0$.

   (ii) Solve $\dfrac{dy}{dx} = \dfrac{x + y + 4}{x - y - 6}$.

   (b) Find a particular solution of $y'' + y = \operatorname{cosec} x$ by method of variation of parameters.

2. (a) Let ϕ_1, ϕ_2 be two solutions of $L(y) = 0$ on an integral *I*, and let x_0 be any point in *I*. Prove that ϕ_1, ϕ_2 are linearly independent on *I* if and only if $W(\phi_1, \phi_2)(x_0) \neq 0$.

   (b) Find the particular solution of $y'' + y' - 6y = e^{-x}$ first by under mined co-efficients and then by variation of parameters.

3. (a) Explain Cauchy's methods of characteristics.

   (b) Find a complete integral of the equation $p^2x + q^2y = z$.

4. (a) Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and solve it.

   (b) Explain separation of variables method.

5. (a) Explain Newton's method.

   (b) Solve the system $10x + y - 2z = 0$, $x + 10y - 3z = 1$ and $-2x + y + 10z = 1$ by using Gauss-Seidal method (3 iterations).

6. (a) Describe the relaxation method.

   (b) State the properties of orthogonal polynomials.

7. Dividing the range into 10 equal parts, find the value of $\int_0^{\pi/2} \sin x \, dx$ by

   (a) Trapezoidal rule. (7)

   (b) Simpson's 1/3 rule. (6)

   (c) Simpson's 3/8 rule. (7)

8. Solve the equation $y'' - 2y' + 2y = e^{2t} \sin t$, $y(0) = -0.4$, $y'(0) = -0.6$ for $y(0.2)$ by R-K method of fourth order.

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DE–3872

Sub. Code
14

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

OPERATIONS RESEARCH

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each questions carries equal marks.

(5 × 20 = 100)

1. (a) Solve : (10)

Maximize $z = x_1 - x_2 + 3x_3$

Subject to the constraints :

$x_1 + x_2 + x_3 \leq 10$
$2x_1 - x_3 \leq 3$
$2x_1 - 2x_2 + 3x_3 \leq 0$
$x_1, x_2, x_3 \geq 0.$

(b) Apply dual simplex method to (10)

Maximize $z = -3x_1 - 2x_2$

Subject to the constraints :

$x_1 + x_2 \geq 1$
$x_1 + x_2 \leq 7$
$x_1 + 2x_2 \geq 10$
$x_2 \leq 3$
$x_1, x_2 \geq 0.$

2. (a) Explain briefly about revised simplex method. (8)

(b) Maximize $z = x_1 + 2x_2$ (12)

Subject to :

$x_1 + 2x_2 \leq 12$

$4x_1 + 3x_2 \leq 14$

$x_1, x_2 \geq 0$ and are integers.

3. (a) Given the product of n numbers $y_1, y_2, y_3 ... y_n = d$ find the minimum value of their sum, using dynamic programming. (10)

(b) Solve the following game. (10)

$$A \quad \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \begin{array}{c} \begin{matrix} \text{I} & \text{II} & \text{III} & \text{IV} \end{matrix} \\ \begin{pmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{pmatrix} \end{array}$$

4. (a) Solve the L.P.P. (10)

Maximize $z = 2x_1 + 5x_2$

Subject to :

$2x_1 + x_2 \leq 43$

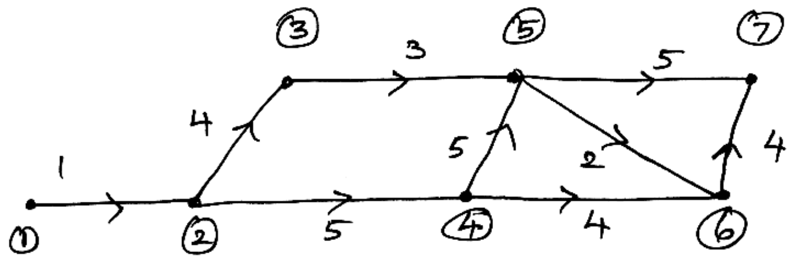
$2x_2 \leq 46$

$x_1, x_2 \geq 0.$

(b) Solve the following game graphically. (10)

$$\begin{array}{c} \\ A_1 \\ A_2 \end{array} \begin{array}{c} \begin{matrix} B_1 & B_2 & B_3 & B_4 \end{matrix} \\ \begin{pmatrix} 2 & 1 & 0 & -2 \\ 1 & 0 & 3 & 2 \end{pmatrix} \end{array}$$

5. (a) Find the critical path for the following network. (8)



(b) Derive Wilson Harris formula for economic lot size problems. (12)

6. (a) A commodity is to be supplied at the rate of 200 units per day. Ordering cost is Rs. 50 and the holding cost is Rs. 2 per unit per day. The delay in supply induces a penalty of Rs. 10 per unit per delay of 1 day. Find the optional policy and the reorder cycle period. (12)

(b) Explain the differences between PERT and CPM. (8)

7. (a) Explain :

(i) Storage cost

(ii) Ordering cost

(iii) Lead time

(iv) Re-order point. (8)

(b) Derive M|M|1 queueing system. (12)

8. (a) A supermarket has 2 sales girls. If the mean service time for each customer is 4 min and the arrival rate is 10 per hour.

Find :

(i) The probability that an arrival has to wait.

(ii) Expected percentage of idle time for each girl. (10)

(b) At a telephone booth, the inter arrival time of telephone calls is 12 min and the average length of a call is 4 min. Find

(i) the probability that a new arrival does not have to wait.

(ii) average number of persons in the system. (10)

DE–3873

Sub. Code
15

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

MATHEMATICAL STATISTICS

Time : Three hours | Maximum : 100 marks

Answer any FIVE questions.

All questions carry equal marks.

$(5 \times 20 = 100)$

1. (a) (i) Define distribution function.

(ii) Let F be a distribution function of a random variable X. Then prove that

(1) F is monotonically increasing

(2) F is continuous from the right

(3) $\lim_{x\to\infty} F(x)=1$ and $\lim_{x\to-\infty} F(x)=0$.

(b) (i) Let $X_1, X_2 ..., X_n$ be a set of independent random variables. Then prove that $Var\left(\sum_{i=1}^{n} a_i X_i\right) = \sum_{i=1}^{n} a_i^2 \operatorname{var}(X_i)$.

(ii) Prove that the first absolute moment about any point is minimum when taken about the median.

2. (a) Let X be a continuous random variable. Then prove that $\sum_{n=1}^{\infty} P(|X| \geq n) \leq E\ |X| \leq 1 + \sum_{n=1}^{\infty} P(|X| \geq n)$.

(b) Two random variable X and Y have the following joint p.d.f

$$f(x, y) = \begin{cases} 2 - x - y, & 0 \leq r \leq 1 \quad 0 \leq y \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find :

(i) Conditional density functions

(ii) $\text{var}(X)$ and $\text{var}(Y)$.

3. (a) State and prove Chebyshev's inequality.

(b) State and prove Central limit theorem.

4. (a) Find out a $100(1-\alpha)\%$ confidence interval for the mean of a normal population having known variance. (7)

(b) Find the confidence interval for the mean of a normal population with unknown variance. (7)

(c) Find the maximum likelihood estimate of the parameter α of a distribution with p.d.f. $f(x) = Cx^{\alpha}, 0 \leq x \leq 1$, Where C is constant. (6)

5. (a) State and prove Neyman-Pearson theorem.

(b) Give the major steps involved in the solution of a testing of hypothesis problem.

6. (a) Obtain the confidence interval for the ratio of variance of two independent normal random samples.

(b) Discuss about moments of multinomial distribution.

7. (a) Explain the single and double sampling plan.

   (b) Discuss about control chart for sample standard deviation.

8. (a) Explain the analysis of variance for one factor of classification.

   (b) Three process A, B and C are tested to see whether their outputs are equivalent. The following observation of output are made :

A	10	12	13	11	10	14	15	13
B	9	11	10	12	13			
C	11	10	15	14	12	13		

   Carry out the analysis of variance and state your conclusion.

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DE–3874

Sub. Code
21

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

COMPLEX ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries equal marks.

(5 × 20 =100)

1. (a) Derive Cauchy-Riemann equations. (8)

   (b) If $u = x^2 - y^2$ is harmonic, find V such that $u + iv$ is analytic. (6)

   (c) Explain with an example the function is continuous does not imply differentiability. (6)

2. (a) Prove that the cross ratio (z_1, z_2, z_3, z_4) is real if and only if the four points lie on a circle or on a straight line. (10)

   (b) State and prove the symmetry principle. (10)

3. (a) State and prove Cauchy's theorem for a circular disk. (10)

   (b) State and prove Morera's theorem. (10)

4. (a) Prove that a non constant analytic function maps open sets onto open sets. (10)

   (b) State and prove Schwarz lemma. (10)

5. (a) State and prove the argument principle. (10)

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} dx$. (10)

6. (a) State and prove Mittag-Leffler theorem. (10)

(b) State and prove Weierstress theorem for entire function. (10)

7. (a) Prove that an elliptic function without poles is a constant. (5)

(b) Prove that the sum of residues of an elliptic function is zero. (5)

(c) Prove that the zeros $a_1, a_2, .., a_n$ and the poles $b_1, b_2, .., b_n$ of an elliptic function satisfy $a_1 + a_2 + ..a_n \equiv b_1 + ... + b_n \pmod{m}$. (10)

8. (a) Show that $p'(z) = -2a \sum_{w} \frac{1}{(z - w)^3}$. (10)

(b) Show that $p(z+u) = -p(z) - p(u) + \frac{1}{4}\left\{\frac{p'(z) - p'(u)}{p(z) - p(u)}\right\}^2$. (10)

DE–3875

Sub. Code
22

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

TOPOLOGY AND FUNCTIONAL ANALYSIS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries equal marks.

(5 × 20 = 100)

1. (a) Let X be topological space and A a subset of X.

   Then prove that (i) $\overline{A} = A \cup D(A)$ and (ii) A is closed if and only if $A \geq D(A)$.

   (b) State and prove Bolzano – Weierstrass theorem.

2. (a) State and prove Lebesgues' covering lemma.

   (b) Prove that the following are equivalent

   (i) X is compact.

   (ii) X is sequentially compact.

   (iii) X has the Bolzano – Weiestrass property.

3. (a) Prove that the product of any non-empty class of Hausdorff space is a Hausdorff space.

   (b) State and prove Uryshon's lemma.

4. (a) If X is a second countable normal space, then prove that there exists a homomorphism f of X onto a subspace of R^{∞} and X is metrizable. (14)

(b) Prove that any continuous image of a connected space is connected. (6)

5. (a) State and prove the four equivalent conditions of a continuous linear transformation. (12)

(b) State and prove Schwarz inequality. (8)

6. (a) State and prove open mapping theorem.

(b) If M and N are closed linear subspaces of a Hilbert space H such that $M \perp N$, than prove that $M + N$ is also closed.

7. (a) State and prove the uniform boundedness theorem.

(b) If $\{e_i\}$ is an orthonormal set in H, and if $x \in H$, then prove that $x - \Sigma(x, e_i)e_i \perp e_j$ for each j.

8. (a) If T is an operator on H for which $(T_x, x) = 0$ for all x then prove that $T = 0$.

(b) Prove that an operator T on H is unitary $\Leftrightarrow$ if is an isometric isomorphism of H onto itself.

DE–3876

Sub. Code
23

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

GRAPH THEORY

Time : Three hours Maximum : 100 marks

($5 \times 20 = 100$)

Answer any FIVE questions.

1. (a) Prove that a graph is bipartite if and only if it contains no add cycle.

   (b) Let G be a graph without loops. Then prove that G is a tree if and only if any two vertices of G are connected by a unique path.

2. (a) Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G.

   (b) Prove that for the complete graph K_n, $\tau(K_n) = n^{n-2}$.

3. (a) Prove that a graph G with more than 2-vertices is 2-connected if and only if any two vertices of G are connected by at least two internally -disjoint paths.

   (b) Prove that if G is a block with $|V(G)| \geq 3$, then any two edges of G lie on a common cycle.

4. (a) State and prove a necessary condition for Hamilton win graphs.

   (b) State and prove Chavatal's theorem.

5. (a) Prove that if G is k-critical , then $\delta(G) \geq k-1$.

(b) Let G be a k-critical graph with a 2-vertex cut $\{u,v\}$. Then prove that $d(u)+d(v) \geq 3k-5$.

6. (a) State and prove Brook's theorem.

(b) For any two integers $k \geq 2$ and $l \geq 2$ prove that $r(k,l) \leq r(k,l-1)+r(k-1,l)$.

7. (a) Prove that the graph $k_{3,3}$ is non-planar.

(b) Prove that a digraph D contains a directed path of length $x-1$ (x is the chromatic number).

8. (a) Let f be a flows and k be a cut in N such that $Valf = capk$. The prove that f is a maximum flour and k is a minimum cut.

(b) State and prove the Max-Flow min-cut theorem.

DE–3877

Sub. Code
24

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

PROGRAMMING IN C/C++

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

Each question carries equal marks.

$(5 \times 20 = 100)$

1. (a) Explain the structure of a C program. (10)

   (b) Describe the various types of data types in C/C++. (10)

2. (a) Explain the gets( ), getchar( ) and scanf functions with examples. (10)

   (b) What do you mean by control codes and arithmetic operators? (10)

3. (a) Discuss the arithmetic operators in C programming with examples. (10)

   (b) Discuss the several types of variables in functions. (10)

4. (a) Explain Nested if statements with an example. (10)

   (b) Write a short note on following : (10)

      (i) USING FLAG statement

      (ii) USING THE break statement.

5. (a) Explain STRINGS in C/C++. (10)

   (b) Define arrays and initialization of arrays in details with examples. (10)

6. (a) Explain structure and functions with suitable examples. (10)

   (b) Define structure and explain how variables and initial values are assigned using structures. (10)

7. (a) Discuss about pointers and functions. (10)

   (b) Explain fprintf( ) and fwrite( ) structures with suitable examples. (10)

8. (a) Write a program to copy the contents of one file into another. (10)

   (b) Write a program to read an entire structure at one time in C/C++. (10)

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DE–3878

Sub. Code
25

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2018.

DISCRETE AND COMBINATORIAL MATHEMATICS

Time : Three hours Maximum : 100 marks

Answer any FIVE questions.

(5 × 20 = 100)

1. (a) Show that the ordinary generating function of the sequence.

$\binom{0}{0}, \binom{2}{1}, \binom{4}{2}, \ldots\ldots, \binom{2r}{r}, \ldots.$ is $(1-4x)^{-1/2}$.

(b) Find the general solution of the recurrence relation $3a_{n+1} - 4a_n = 0, n \geq 0; a_1 = 5$.

2. (a) Solve the difference equation $a_n + 2a_{n-1} = n + 3$; $a_0 = 3$.

(b) Using the technique of generating function, solve the relation $a_n = a_{n-1} + 2(n-1); a_0 = 2$.

3. (a) Derive the principle of inclusion and exclusion.

(b) Obtain the number of derangements d_n of n objects, using a recurrence relation.

4. (a) Find the number of permutations of letters a, b, c, d, e and f in which neither the pattern ace nor the pattern fd appears.

(b) Find the number of permutations of the letters $\alpha, \alpha, \beta, \beta, \gamma$ and γ so that no α appears in the first and second positions, no β appears in the third position, and no γ appears in the fifth and sixth positions.

5. (a) State and prove Burnside lenma.

(b) Explain polya's fundamental theorem.

6. (a) Find the cycle index of the permutation group s_4, consisting all the permutations on $\{1, 2, 3, 4\}$.

(b) Find the number of ways of painting the four faces a, b, c and d of the pyramid with two colours x and y.

7. (a) Prove that in an arbitrary lattice $(l, \leq)$ the following inequalities are valid.

$x \vee (y \wedge z) \leq (x \vee y) \wedge (x \wedge z)$

$x \wedge (y \vee z) \leq (x \wedge y) \vee (x \wedge z) \ \forall x, y, z \in L$.

(b) Prove that a lattice L is modular if and only if for all $x, y, z \in L$, $x \vee (y \wedge (x \vee z)) = (x \vee y) \wedge (x \vee z)$.

8. (a) Show that the product of two distributive lattice is a distributive lattice.

(b) Explain Karnaugh map with examples.

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