(Pages : 4) **B – 3083**

Reg. No. : ....

Name : ....

School of Distance Education

First Semester B.Sc. Degree Examination, December 2016
First Degree Programme under CBCSS
MATHEMATICS
Core Course
MM 1141 : Methods of Mathematics
(2017 Admission Onwards)

Time : 3 Hours Max. Marks : 80

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Give two numbers that are coprime.
2. Define Fermat number.
3. Using Euclidean Algorithm calculate gcd (272, 1479).
4. Find the smallest number ≥ 0 which is congruent to 31 (mod 35).
5. Examine whether the circle $x^2 + y^2 = 1$ is a graph of some function of x.
6. Find $\lim_{x \to 0} f(x)$, where $f(x) = \frac{x}{|x|}, x \neq 0$.
7. Find $\frac{dy}{dx}$ where $y = x^2 \tan x$.
8. Use implicit differentiation to find dy/dx if $6y^2 + \cos y = x^2$.
9. Give the focus and directrix of the parabola $x^2 = 4py$ $(p > 0)$.
10. Give the polar equation of a conic for which directrix is the line $x = -d$ to the left of the pole, e is the eccentricity and origin is a focus. **(10×1=10 Marks)**

P.T.O.

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Using Induction, prove that for each $n \in \mathbb{N}$, the sum of the first n natural numbers is given by $1 + 2 + ... + n = \frac{1}{2}n(n+1)$.

12. Prove that $\sqrt{2}$ is irrational.

13. Use the Euclidean Algorithm to obtain integers x and y satisfying the equation gcd (56, 72) = 56 x + 72y.

14. Prove that 11 divides a if and only if 11 divides the alternating sum of the digits of a.

15. If $m \neq n$, then prove that the Fermat numbers F(m) and F(n) are relatively prime.

16. Prove that if $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$ then $a \equiv c \pmod{n}$.

17. Find $\lim\limits_{x \to -\infty} \frac{\sqrt{x^2+3}}{5x-6}$.

18. Using Intermediate value theorem, solve $x^3 - 9x + 1 = 0$ for a root between x = 2 and x = 4.

19. Let $y = x^2 + 1$.

    a) Find the average rate of change of y with respect to x over the interval [3, 5].

    b) Find the instantaneous rate of change of y with respect to x when x = –4.

20. If x^0 denotes the angle x measured in degrees, then find $\frac{d^2}{dx^2} \sin(x^0)$.

21. Find the slope of the curve $y = \sqrt{x}$ at x = 16.

22. State the reflection property of hyperbolas. **(8×2=16 Marks)**

Answer **any 6** questions from the questions **23** to **31**. These questions carry **4** marks **each**.

23. Show that square of any odd integer is of the form 8k + 1.

24. Find all solutions of 32 = 365x + 1876y.

25. Prove that if p is a prime and p divides ab, then p divides a or p divides b.

26. Find the domain and range of $f(x) = \frac{x+1}{x-1}$.

27. The position p(x, y) of a particle moving in the xy – plane is given by the equations and parameter interval $x = \sqrt{t}, \quad y = t, \quad t \geq 0$. Identify the path traced by the particle and describe the motion.

28. Describe the vertical asymptotes of y = sec x.

29. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 1 ft/s. How fast is the area of the spill increasing when the radius of the spill is 20 ft ?

30. Sketch the graph of the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ showing the foci.

31. Find the equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes $y = \pm \frac{4}{3}x$.

(6×4=24 Marks)

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. a) State and prove Euclid's algorithm.

    b) Given integers a, b, e, there are integers m and n with am + bn = e if and only if (a, b) divides e.

33. a) Sketch the graph of the functions $y = \sqrt{x} - 5$ and $y = \sqrt{x} + 5$.

b) A wheel of radius a rolls along a horizontal straight line. Find parametric equations for the path traced by a point P on the wheel's circumference.

34. a) Suppose that a car moves with a constant velocity of 73 ft/s in the positive direction of an s-axis. Given that the s-coordinate of the car at time t = 0 is s = 55. Find an equation for s as a function of t, and graph the position versus time curve.

b) Find $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$.

35. a) Suppose that the axes of an xy – coordinate system are rotated through an angle of $\theta = 45^\circ$ to obtain an $x'y'$ – coordinate system. Find the equation of the curve $x^2 - xy + y^2 - 6 = 0$ in $x'y'$ – coordinates.

b) Identify and sketch the cruve $xy = 1$. **(2×15=30 Marks)**
